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## RESEARCH MEMORANDUM

ANALYSIS OF STAGE MATCHING AND OFF-DESIGN PERFORMANCE  
OF MULTISTAGE AXIAL-FLOW COMPRESSORS

By Harold B. Finger and James F. Dugan, Jr.

Lewis Flight Propulsion Laboratory  
Cleveland, Ohio

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NATIONAL ADVISORY COMMITTEE  
FOR AERONAUTICS

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## NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

RESEARCH MEMORANDUMANALYSIS OF STAGE MATCHING AND OFF-DESIGN PERFORMANCE OF  
MULTISTAGE AXIAL-FLOW COMPRESSORS

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## SUMMARY

An analysis is presented to give a qualitative picture of the operation of each stage in a high-pressure-ratio multistage compressor over a full range of operating flows and speeds and to point out methods of improving off-design performance. The analysis used is based on single-stage performance results that have been "stacked" to form a multistage compressor in which the design or match point of each stage has been arbitrarily selected. Stage interaction effects and changes of stage efficiency with rotor speed are neglected in this method and the temperature rise across each stage is assumed to vary directly as the blade speed squared.

The analysis shows that a high-pressure-ratio compressor, so designed that all stages operate at peak efficiency at design conditions, operates at low efficiencies when the compressor is operated at low speed. Higher efficiencies can be obtained at low-speed operation by so designing the compressor that all stages operate at peak efficiency at a speed below design speed; that is, the inlet stages operate at less than the optimum angle of attack at design speed and the exit stages at greater than the optimum angle of attack. Improvements in off-design performance can also be accomplished by setting stator-blade angles in a given compressor to load the exit stages and unload the inlet stages. As was to be expected, flat stage-pressure-ratio and efficiency curves give better multistage-performance characteristics than peaked stage curves. Stages having a weight flow for peak pressure ratio considerably lower than the weight flow for peak efficiency result in a compressor with an improved acceleration margin.

The analysis has indicated that in every compressor, there is a stage which operates over a minimum angle of attack range at all flows and speeds. On the basis of the analysis, therefore, this section of the compressor may be highly loaded.

## INTRODUCTION

The trend toward increased compressor-pressure ratios aimed at increasing the fuel economy of turbojet engines having high turbine-inlet temperatures has introduced serious problems relating to the off-design-speed performance of the engine. At speeds below design, it has been found that the operating line of the engine often passes close to or through the compressor surge region, making rapid acceleration and sometimes even stable operation impossible. Below 50 percent of design speed, engine starting problems are introduced by low compressor efficiencies and the surge-line location. Both of these problems are associated with the matching of the stages in the multistage unit. The higher the over-all compressor-pressure ratio, the more consideration must be given to means of matching stages for improved off-design-speed operation.

General discussions of the problems related to stage matching and to the effects on stage performance of off-design operation are presented in references 1 and 2. The discussion in reference 2 indicates the compromise which must be made to obtain good engine-starting characteristics as well as good design-point operation. A one-dimensional analysis similar to that used in the present report is applied in reference 3 to the determination of the effect of stage loading on range of operation in a multistage compressor. The analysis of reference 3 is based on single-stage and two-dimensional cascade data and on the assumption that surge of the compressor occurred when any given blade row stalled. A similar analysis is presented in reference 4. Both analyses indicate that the use of highly loaded stages in a multistage compressor will give better off-design-speed performance than lightly loaded stages because of the smaller departure from the design axial velocity across each stage.

The present report is intended to give a qualitative picture of the operation of each stage of a multistage compressor over a range of actual operating conditions from 50 to 100 percent design speed and to point out some means of achieving improved off-design performance. The method used in the report is primarily analytical and is based on single-stage-performance results. The performance curves of the single-stage unit have been "stacked" to form a multistage compressor in which the design or match point of each stage has been arbitrarily selected. This stacking procedure is described in detail. The final results obtained by this procedure are qualitative in that they point to improved off-design performance but do not contain numerical values for the improvements obtained or the design changes required.

Consideration is given first to the effect of the single-stage performance on the over-all performance of the unit, then to the various means of matching the stages of a multistage compressor, and finally to loading and unloading the stages in a given compressor by adjusting the stator-blade angles.

## SYMBOLS

The following symbols are used in this report:

A	area
$c_p$	specific heat at constant pressure
g	acceleration of gravity
J	mechanical equivalent of heat
K	proportionality constant
$K'$	$(\tan \beta_R + \tan \alpha_S)/K$
P	total pressure
p	static pressure
T	total temperature
U	blade speed
V	absolute velocity
$V_z$	axial component of absolute velocity
$V_\theta$	tangential component of absolute velocity
$V'$	resultant relative velocity
$V'_\theta$	tangential component of relative velocity
W	weight flow
$\alpha$	absolute flow angle
$\beta$	relative flow angle
$\gamma$	ratio of specific heats
$\delta$	ratio of total pressure to standard sea-level pressure
$\eta$	adiabatic efficiency
$\theta$	ratio of total temperature to standard sea-level temperature

$\rho$  density

$\phi$  flow parameter

Subscripts:

i compressor inlet

o compressor outlet

R stator inlet, rotor outlet

S rotor inlet, stator outlet

1,2,3 . . . n inlet to first, second, third . . . any arbitrary stage

#### METHOD OF ANALYSIS

The determination of the over-all performance of a compressor depends on the knowledge of the performance of each stage of the compressor. Because it has been found that two- and three-dimensional treatments are generally difficult to handle, especially when sections of a blade are stalled, an essentially one-dimensional solution of the matching problem is used in this report.

The method of analysis is based upon the performance of a typical single-stage compressor which is taken to be representative of the stages throughout the multistage compressor being analyzed. The single-stage-performance curves are "stacked" to form the multistage unit by making the performance variables independent of compressor size and assuming that the mean radius is constant. The performance of the multistage compressor is then determined over a wide range of speeds and flows by stacking.

In applying the single-stage-performance curves to the design of the multistage compressor, the following assumptions have been made:

(1) The single-stage-efficiency curve used represents the efficiency of each stage of the compressor regardless of rotor speed.

(2) Each stage in the compressor operates along the specified single-stage-performance curves; thus any stage interaction effects are neglected.

(3) The effect of changes in the ratio of stagnation to static density is neglected throughout the analysis.

(4) The effects of loading on attainable stage efficiency and range of good efficiency are negligible.

Such effects are believed to vary primarily with the method of loading (increasing camber or Mach number) and the velocity diagrams used. Because only limited data are now available on these effects, consideration cannot be given to them in this analysis.

2535 The first assumption requires that the change in efficiency with speed be negligible. For a single stage the peak efficiency is generally fairly constant, varying only about 3 percent with changes in speed from 50 percent to 100 percent of design speed. However, there is a noticeable effect of speed on the efficient range of operation of a single stage. By the second assumption, any effects of one stage on the performance of the preceding or following ones are neglected. It is known that such effects do exist; however, because these effects are believed to vary with the position of the stage in the compressor as well as with the stage type, the stage interaction effects cannot be accurately considered in the analysis until more complete experimental data become available. Such interaction effects must be evaluated if various types of stage design are to be evaluated for optimum multistage-compressor design. The third assumption of constant density ratio requires a constant Mach number throughout the compressor over the entire range of speeds considered. For the range of Mach numbers (approximately 0.5 to 0.8) encountered in the analysis of the performance of a given one-dimensional compressor design, the maximum error in the value of the velocity ratio used as an indication of angle of attack would be approximately 8 percent. These errors tend to make the results of the present analysis conservative insofar as the required operating range of a stage is concerned. Although the error over the range of speeds for a given compressor design may be large, it is expected that the error in any given stage operating at a specified speed and inlet flow will be essentially constant for all of the designs considered. Therefore, qualitative comparison of the performance of the different designs investigated will be valid.

### Single-Stage Performance

The single-stage performance used as the basis of the present investigation was determined experimentally. Early performance tests of the stage used are reported in reference 5. The performance of this stage at 50 percent of design speed over a wide range of equivalent weight flow is presented in figure 1 where stage pressure ratio and adiabatic efficiency are plotted against equivalent weight flow. The maximum efficiency attained at the 50 percent speed is comparatively low, only 86 percent. Thus the efficiency of the multistage compressor will be low at all speeds when this single-stage-efficiency curve is

used as the basic stage performance. It is important to note that the pressure ratio remains essentially constant at low flows. An explanation for this phenomenon is indicated in figure 2 which shows a longitudinal section through the compressor. It is believed that a rotor tip-stall condition at these low flows sets up a recirculating flow, such as that described in reference 6, which increases in size as flow is reduced. Thus the decrease in flow, accomplished by a reduction in the effective flow area, causes the hub stations to continue to operate at essentially constant flow angles. As a result, the mass-averaged pressure ratio remains almost constant.

The single-stage-performance curves presented in figure 1 are not suitable for the stacking method used in the present paper because the variables involved are a function of compressor size and speed. These variables must be independent of size in order to apply equally well to all stages of the compressor. The desired parameters are obtained from Euler's turbine equation, the continuity condition, and the velocity-diagram geometry presented in figure 3.

The temperature rise across the stage is related to the velocity diagram by Euler's turbine equation; no change in radius is assumed

$$c_p J g \Delta T = U \Delta V_\theta$$

If the axial velocity is assumed to be constant across the rotor row:

$$c_p J g \Delta T = U V_z (\tan \alpha_R - \tan \alpha_S) \quad (1)$$

but

$$\tan \alpha_R = \frac{U - V'_z}{V_z} = \frac{U}{V_z} - \tan \beta_R$$

If the expression for  $\tan \alpha_R$  is substituted into equation (1) and the terms are rearranged,

$$\frac{c_p J g \Delta T}{U^2} = 1 - \frac{V_z}{U} (\tan \beta_R + \tan \alpha_S) \quad (2)$$

Equation (2) indicates that if  $\alpha_S$  and  $\beta_R$  are constant, which is approximately the case for unstalled blade rows, then the ratio of temperature rise to mean blade speed squared is constant at a fixed value of  $V_z/U$ . A curve of  $\Delta T/U^2$  against  $V_z/U$  obtained from single-stage data is therefore assumed to be constant regardless of rotor speed. Although the terms of equation (2) are dimensionless and appear to be satisfactory for the stacking procedure, the variables involved must be expressed in terms of the performance parameters of figure 1.

The left side of equation (2) is related to the stage pressure ratio and efficiency by the following expression:

$$\frac{c_p J g \Delta T}{U^2} = c_p J g \frac{T_{n-1}}{U^2 \eta} \left[ \left( \frac{P_n}{P_{n-1}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \quad (3)$$

The term  $V_z/U$  for a given stage can be expressed in terms of equivalent weight flow and equivalent rotor speed by application of the continuity condition:

$$W = \rho g A V_z$$

or

$$\frac{W}{U} \frac{1}{A} = \rho g \frac{V_z}{U} \quad (4)$$

If the ratio of static to stagnation density is assumed to be constant throughout each of the compressors analyzed, equation (4) for the first stage becomes

$$\phi = \frac{W \sqrt{\theta_1} / \delta_1}{U / \sqrt{\theta_1}} \frac{1}{A_1} = K \frac{V_{z,1}}{U} \quad (5a)$$

or for any stage  $n$

$$\phi = \frac{W \sqrt{\theta_1} / \delta_1}{U / \sqrt{\theta_1}} \frac{1}{A_1} \frac{T_n}{T_1} \frac{P_1}{P_n} \frac{A_1}{A_n} = K \frac{V_{z,n}}{U} \quad (5b)$$

The term on the left side of equations (5a) and (5b) is referred to in the present report as the flow parameter. The constant  $K$ , whose value is not required for the analysis, includes the proportionality constant relating the static and stagnation densities and the standard sea-level pressure and temperature. For the present case,  $A_1$  and the design value of  $U / \sqrt{\theta_1}$  are taken to be equal to those of the experimental single-stage compressor already discussed. These values are 1.069 square feet frontal area and 825 feet per second, respectively.

If the relations of equations (3) and (5) are substituted into equation (2), the expression for the single-stage temperature rise in terms of the conventional performance variables becomes

$$\frac{c_p J g \Delta T}{U^2} = c_p J g \frac{T_{n-1}}{U^2 \eta} \left[ \left( \frac{P_n}{P_{n-1}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] = 1 - \frac{\phi}{K} (\tan \beta_R + \tan \alpha_g)$$



However, because  $\alpha_S$  and  $\beta_R$  have been assumed constant

$$\frac{c_p J g \Delta T}{U^2} = c_p J g \frac{T_{n-1}}{U^2 \eta} \left[ \left( \frac{P_n}{P_{n-1}} \right)^{\frac{r-1}{r}} - 1 \right] = 1 - K' \phi \quad (6)$$

Equation (6) indicates that a curve of  $\Delta T/U^2$  against  $\phi$  can be assumed to be constant regardless of rotor speed. Correlation of compressor-stage data obtained in tests of an engine operating at various speeds and also in compressor component tests have indicated that the effects of variations in  $K'$  are actually small and will have negligible effects on the comparisons made herein.

The single-stage performance, in terms of the flow parameter, is presented in figure 4 where the design-speed stage temperature rise and adiabatic efficiency of the stage are plotted against the inlet-flow parameter (equation (5a)). These curves are used as the working curves to design the multistage compressor for the present analysis. Because the temperature-rise curve (fig. 4(a)) has been presented at the design speed, the temperature rise at any speed less than design can be determined by multiplying  $\Delta T$  of figure 4(a) by the square of the percent of design speed. The efficiency curve is required to permit the determination of the pressure ratio across any stage.

#### Multistage-Compressor Design

Ordinarily, the design of a multistage compressor requires that the weight flow, the type of velocity diagram used in each stage, and the desired over-all pressure ratio be specified. These independent variables in combination with theoretical or empirical design limits determine the work to be obtained from each stage, the number of stages, and the effective flow area required. The geometric area is then often adjusted to take into account the growth of annulus boundary layer through the compressor. In a one-dimensional analysis such as the present one, based on known single-stage performance curves, it is necessary only to specify the design point of each stage of the compressor on the stage-performance curve (fig. 4) and from this condition to compute the required effective annular-flow-area variation through the unit.

Example of procedure. - In the present analysis a compressor will be designed for an over-all pressure ratio of approximately 10. For the present example, which is the method of design I, the area variation required to permit each stage to operate at its peak stage-efficiency point at design speed was determined. This condition is specified as

the design condition. It is required that each stage operate at a flow parameter of 0.0260 (fig. 4(b)) at the compressor design point. For this assumption,  $V_{z,1} = V_{z,n}$  in equation (5) because  $U$  is assumed constant through the compressor.

If the first stage operates at a flow parameter of 0.026, the value of  $\Delta T$  is 37.0 according to figure 4(a) and the efficiency is given as 86 percent in figure 4(b). If these values are substituted into equation (3), the pressure ratio across the first stage can be determined. Because the design requirement specifies that all stages operate at the same flow parameter, substitution of the pressure ratio and temperature ratio in equation (5b) permits the determination of the area ratio required across the first stage. The same procedure is then repeated for the second stage and on through the compressor. For this particular design, referred to as the maximum design-speed efficiency case, 16 stages were required to give a pressure ratio of 10. The area variation determined for this design by the method outlined is presented in figure 5 as the ratio of the inlet area of any stage to the compressor-inlet annular area against the stage number. The convergence of the area is typical of axial-flow compressors; this convergence increases with the over-all pressure ratio.

Once the area variation is fixed by the design stage-matching specifications, it is possible to determine the operating point of each stage in the compressor for any specified inlet flow and speed. For instance, if it is assumed that the first stage is operating at a flow parameter of 0.02 at design speed, the temperature ratio and pressure ratio for the first stage can be computed and substituted into equation (5b); the flow parameter for the second stage can then be determined. With this value of flow parameter, the performance of the second stage at the specified inlet flow can be determined and the flow parameter for the third stage can be computed. The same procedure is followed throughout the compressor. The over-all performance of the compressor can be computed from the stage pressure ratios and temperature rises. If the speed of the unit is varied in addition to the flow parameter for the first stage, it is necessary only to multiply the curve of design-speed temperature rise (fig. 4(a)) by the square of the ratio of blade speed to design blade speed to determine the performance of each stage at the desired speed. The same procedure is then used to determine the over-all compressor performance at the new speed.

#### Compressor Designs Investigated

By use of the method described, three groups of compressors were investigated to obtain an indication of how improved off-design-speed performance can be achieved and to give a qualitative picture of the operation of each stage in a high-pressure-ratio multistage compressor

over a full range of flows and speeds. In the first group of compressors, the effect of stage performance on the off-design performance of the compressor was considered. In the second group, qualitative indications of the effects of designing compressors for different stage-matching points were studied. The effect on over-all performance of loading exit stages and unloading inlet stages by stator-blade-angle resetting was studied in the third group.

## DISCUSSION OF RESULTS

### Effect of Stage-Performance

Design I. - Design I was discussed in the example of the method of computation and is based directly on single-stage performance data obtained experimentally. All of the stages were set to operate at the peak stage-efficiency point at the design speed and the area variation (fig. 5) was computed accordingly. Such a procedure is used in designing most commercial compressors because engine performance must be guaranteed at the design point and manufacturers have generally been reluctant to sacrifice design-point performance for improved off-design performance. The performance of design I will be compared with that of the other designs to show what changes in off-design-speed performance can be expected.

The over-all performance of design I is shown in figure 6 by curves of total-pressure ratio and adiabatic efficiency against equivalent weight flow for constant speeds from 50 to 100 percent of design speed. In this analysis, the surge line is taken to be the line through the peak pressure ratios at the various speeds. This assumption was thought to be satisfactory on the basis of the analyses of references 7 and 8 which indicate the occurrence of surge near the peak pressure-ratio point. Extension of the performance curves to the maximum flows at low speeds was impossible because flows higher than those presented resulted in choking in the outlet stage of the compressor. The occurrence of choking in any stage makes it impossible to determine the operating point for that stage by the method used in the present report as the performance curve becomes vertical in the choke region (fig. 4(b)). Because of this limitation on the calculations, the peak efficiency at the low speeds was not determined. Increases in efficiency above the maximum shown at each speed are believed to be small, however. The maximum efficiency shown is taken to be the peak efficiency at each speed because at higher flows the exit stages operate in a very low-efficiency region to the right of the curve shown in figure 4(b) and lower over-all efficiency results. The maximum efficiency at design speed is approximately 80 percent at a weight flow of approximately 23 pounds per second and a pressure ratio of approximately 10 (fig. 6). The maximum efficiency decreased from about 80 percent at design speed to approximately 56 percent at 50 percent design speed.

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This drop-off in efficiency with reduced speed is explained in figure 7 where the stage pressure ratio is plotted against the flow parameter for constant percentages of inlet-stage design equivalent speed. The broken lines indicate the operating range of the first, eighth, twelfth, and sixteenth stages for maximum compressor-pressure-ratio conditions over the speed range. The circular symbols represent stage operation at an over-all speed equal to design speed and the squares indicate operation at 50 percent of design. The equivalent weight flows for maximum stage efficiency and for maximum pressure ratio are practically the same (see fig. 1).

Figure 7 shows, therefore, that the first stage operates at approximately peak efficiency at design speed (shown by circle at intersection of stage 1 curve and 100-percent-speed curve). As speed decreases, the flow parameter for stage 1 decreases and the angle of attack increases. The increasing angle of attack leads to lower efficiency and eventually to high-angle-of-attack blade stall. At 50 percent speed, stage 1 operates at a very low flow parameter. The eighth stage operates over a much narrower range of flow parameter and angle of attack. Because of the temperature rise up to the eighth stage, this stage operates at an equivalent speed below design speed, which results in a reduced stage pressure ratio. Stage twelve was found to operate over the minimum range of angle of attack; this suggests that in such a compressor the twelfth stage could be highly loaded. If loading has no effect on range, however, from range considerations alone all stages can be highly loaded. The position of the stage operating over the smallest range of angle of attack is determined by the stage performance curves and the method of design. The sixteenth stage operates at approximately peak efficiency at design speed, but as speed decreases the flow parameter increases and the angle of attack decreases. This decreasing angle of attack leads to negative-angle-of-attack blade stall and low efficiency. Stage pressure ratio and stage adiabatic efficiency are plotted against stage flow parameter in figure 8. The efficiency curve shows the relation of the stage operating point to the peak efficiency. Constant compressor-speed lines (solid) and stage-operation lines (dashed) for maximum over-all compressor-pressure-ratio conditions are also presented. Figures 7 and 8 show that the decrease in equivalent speed through the compressor results in a decrease in stage pressure ratio; that the first stages of the compressor operate at low efficiencies at low speeds; and that the exit stage operates on the low angle-of-attack side of the peak-efficiency point, thereby resulting in a marked decrease in over-all efficiency of the compressor as speed is reduced from design.

Design II. - In design II, all of the stages were designed to operate at the peak stage-efficiency point at design speed. However, in this case, the  $\Delta T$  curve of design I is used in conjunction with the stage-pressure-ratio curves presented in figure 9 as dashed lines. The difference is in the low-flow portion of the curves only. Whereas the

design-I curve remained practically flat at the low flows, the present curve decreases gradually to the left of the peak pressure-ratio point. Thus the efficiency curve is altered as shown in figure 10 in which the standard efficiency curve is presented as the solid line and the modified efficiency curve is presented as the dashed line. The efficiency curve for the modified stage drops off more rapidly than the original curve in the low-flow region. Because the modifications were made only in these low-flow regions, the area variation which is calculated at the peak efficiency point for design II is again the same as that required of design I (fig. 5).

A comparison between the over-all performance of designs I and II is presented in figure 11. Design II exhibits lower efficiencies and pressure ratios than design I at all speeds except design. At design speed, the performances are the same since only the low-flow portions of the stage curves were altered. The surge line of design II lies below the surge line of design I and the efficiencies for design II are appreciably less than those of design I at the low speeds. The operating conditions of design II at peak over-all compressor-pressure-ratio flow conditions are shown in figure 12. Comparison of figures 12 and 8 indicates that the stages operate over practically the same flow ranges in both designs. In design II, stage 11 was found to operate over the minimum angle-of-attack range. The differences in the performance maps of designs I and II can be explained by the differences in the stage curves in each design. The stage curves for design II exhibit lower pressure ratios and lower efficiencies at low flow parameters than the stage curves of design I. Hence, design II results in lower over-all pressure ratios and lower over-all efficiencies at speeds below design than does design I. As was expected, therefore, the flatter pressure-ratio and stage efficiency curves will give improved over-all performance characteristics at off-design conditions.

Design III. - All the stages of design III were designed to operate at the peak stage-efficiency point at design speed. However, the efficiency curve of design I (fig. 4(b)) was used in conjunction with the stage-pressure-ratio curve presented in figure 13. The temperature-rise curve (fig. 4(a)) was modified accordingly. The curve of design I has been so modified that the peak pressure ratio now occurs at a reduced value of flow parameter. For this design, the peak stage efficiency occurs at a flow parameter of 0.026 whereas peak stage pressure ratio occurs at a flow parameter of 0.0175. Because designs I and III have the same design points the area variation is the same as for design I.

The over-all performance of design III is compared with that of design I in figure 14. The effect of causing the peak stage pressure ratio to occur at a lower flow (design III) while the peak efficiency or design point remains fixed was to reduce the weight flow for peak over-all compressor-pressure ratio at the high speeds and to increase

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both the peak pressure ratio and weight flow at the low flows. These changes in over-all performance caused the surge line for design III to lie at lower flows than the surge line for design I. The increase in peak over-all pressure ratio is the result of the increased stage pressure ratio shown in figure 13 for design III. The maximum shift in the surge line occurred at the high compressor speeds. Improvements in this range of speed are not generally necessary for improved acceleration or operating characteristics. However, some improvement in the surge line position results at the low speeds. The peak efficiencies for both designs are essentially equal over the entire range of speeds investigated. Because acceleration of an engine requires an increase in turbine-inlet temperature, which causes the operating line of the compressor to be shifted closer to the surge line, the shift in the surge line to the lower flows encountered in design III will permit a wider margin for acceleration.

#### Effect of Variation of Stage Match Points

Design IV. - In design IV, each stage was set to take full advantage of its efficient range of angle of attack by operating at a different value of flow parameter or angle of attack at the design speed as indicated in figure 15. The first stage was set to operate to the right, or high flow and low angle of attack, side of the peak efficiency point; the exit stage was set to operate at the low-flow or high-angle-of-attack side of the peak efficiency point at design speed. The intermediate stages were set between these limits such that the middle stage operated at the peak efficiency point. The efficiency curve was the same as that used for design I but the  $\Delta T$  curve was altered by multiplication by a constant factor greater than 1.0 all along the curve so that compressor total-pressure ratio (10:1) obtained at the design point (flow of 23.1 lb/sec) in design I was duplicated in the present design. The over-all performance of design IV is presented in figure 16 as curves of total-pressure ratio and adiabatic efficiency against equivalent weight flow for constant speeds of 50 to 100 percent design speed. Because of the increased design flow parameter in the first stage (fig. 16), design IV is characterized by higher weight flows than design I; the point of 23.1 pounds per second and the pressure ratio of 10 are therefore in the surge region.

To match the peak design-speed pressure ratios of designs IV and I, the stages of design IV should be more highly loaded; to match the weight flows at these points, the diameter of design IV could be reduced or the blade-setting angles could be modified. In order that designs IV and I may be compared as they are, their performance maps are presented in figure 17 as curves of adiabatic efficiency and percent of maximum pressure ratio against percent of weight flow at maximum pressure ratio. The surge line for design IV lies slightly below the surge line for

design I. The low-speed efficiencies of design IV are considerably higher than those of design I, but the design-speed efficiency is sacrificed slightly. The peak efficiency for design IV varies only from 73 percent at 50 percent of design speed to a maximum of 81 percent at 80 percent speed.

The operating characteristics of stages of design IV are presented in figure 18 for peak over-all compressor-pressure-ratio flow conditions at various speeds. The stages operate over flow ranges which do not vary as much from the flow for peak efficiency as was the case in design I (fig. 8). Hence, all stages operate in a better efficiency range than design I at all speeds from 50 to 100 percent of design speed; the resulting small variation of peak efficiency with speed is indicated in figure 17. In the present modification, stage nine was found to operate over the minimum range of angle of attack. At 80 percent speed, all the stages of design IV operate near the maximum efficiency point. When the stage match points at design conditions were suitably spread, all the stages could be made to operate at peak efficiency at any specified speed below 100 percent design speed. In the present case, a compressor almost identical with design IV would have resulted if all the stages had been designed to operate at peak efficiency at 80 percent design speed. This method of design is not practical, however, because it would be difficult to predict a design-speed weight flow and pressure ratio and to determine the aerodynamic design limits to be used.

#### Effects of Loading and Unloading Stages

Sometimes a compressor exhibits poor off-design performance; this might occur in a compressor designed by the method of design I. If complete redesign of the unit by a design method such as that described for design IV is not feasible, then some simpler method must be applied to permit an improvement in performance. It was, therefore, decided to determine the suitability of improving off-design performance of high-pressure-ratio multistage compressors by use of adjustable stator blades, which would approximate the design of IV without the area redesign. In the discussion of design I, it was noted that at low speeds, the inlet stages operated at excessively high angles of attack and the exit stages operated on the low-angle-of-attack side of peak stage efficiency. Thus, adjustment of the inlet stages to cause them to operate at lower angles of attack at low speeds should result in an improvement in part-speed performance. Similarly, increasing the angle of attack in the exit stages should result in improved performance in these stages. In the present analysis, the loading and unloading were accomplished by stator-blade-angle adjustment. This method of attack has been applied experimentally and has shown that the qualitative results of the present analysis are accurate. Unloading a specified rotor would require that

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the preceding stator-setting angle, as measured from the axial direction, be increased (fig. 3). Equation (2) indicates that for a given value of flow parameter, an increase in flow angle  $\alpha_g$  leaving the stator requires that the temperature-rise function decrease. Thus, unloading a stage by adjustment in the setting angle of the preceding stator row can be approximated in the present analysis by shifting the stage-performance curves (fig. 4) to the left along the flow-parameter scale or to lower values of flow parameter. It is assumed in the analysis that the effect of the change in stagger on performance is negligible.

Design V. - The effect of unloading the inlet stages of an existing compressor was determined by shifting the stage-performance curves for design I 0.001 on the flow-parameter scale toward reduced values of flow. This shift corresponds to a change in angle of attack on the following rotor of approximately  $1.25^\circ$ . With the exception of the shift along the flow scale, the stage-performance curves (fig. 4) remained identical with those used in the original design computations. It was assumed that the first five stages of the compressor would operate along the unloaded stage-performance curves and the remainder of the compressor would be composed of the standard stages of design I. The area variation previously computed for design I was used in the present computations.

Operation of the inlet stages at reduced angles of attack at low speeds should result in improved efficiency and increased pressure ratios at the low speeds. The performance of design V is compared in figure 19 with that of design I. Design V exhibits higher pressure ratios, efficiencies, and weight flows than design I for speeds up to 90 percent of design speed. For design speed, the efficiencies of the two designs are the same and the pressure ratios and weight flows of design V are lower than those of design I. The design with unloaded inlet stages would have better acceleration characteristics than design I even though the surge line remained essentially unchanged because the off-design-speed efficiencies are higher. The decreased pressure ratios and weight flows at design speed are caused by the shifts in the stage performance to reduced flow. For a given flow at design speed, the pressure ratios of the individual stages in the designs with the unloaded inlets are reduced, thereby reducing the total pressure ratio. The decreased weight flows result from the fact that the exit stages operate at high negative angles of attack (resulting in separation and a choking condition) for a lower weight flow in design V with the unloaded inlet stages than in design I. The increased pressure ratios, weight flows, and efficiencies at part speed can also be explained by the shifts in the stage-performance curves to reduced values of flow. At part speed, the inlet stages operate at high angles of attack or low values of flow parameter. In this range, the stage-performance curves for design V give higher values of pressure ratio and



efficiency, thus resulting in higher over-all pressure ratio and efficiency values. The increased weight flows at part-speed operation result from the fact the exit stages operate at negative angles of incidence for a higher weight flow in the design with the unloaded inlet stages.

Design VI. - In addition to the effect of adjusting stator-blade angles so as to unload the inlet stages, it was desired to determine the effect of loading exit stages, also by stator-blade adjustment. This loading was accomplished in the present design by shifting the exit-stage performance curves of design I along the flow-parameter scale by 0.001 to higher values of flow. Equation (2) indicates that this is the direction required for loading a stage. The computations were made on the basis of the area variation determined in design I with the assumption that the last five stages were loaded by stator adjustment; the remaining stages were the standard ones used in design I. Loading the exit stages of a compressor the exit stages of which operate at excessively low angles of attack at low speeds should improve the part-speed performance by increasing efficiency and pressure ratio. The performances of designs VI and I are presented in figure 20. The low-speed efficiencies, pressure ratios, and weight flows of design VI are slightly higher than those of design I. Therefore, design VI should have somewhat better acceleration characteristics than design I even though the surge line is unaltered. The design speed performances of designs VI and I are practically the same.

#### SUMMARY OF RESULTS

The following results were obtained from a simplified theoretical analysis of various design methods to obtain improved off-design performance of multistage compressors:

1. Low over-all efficiency for off-design operation exists in a compressor so designed that all stages operate at peak efficiency at the design point. This low efficiency is caused by the first stages operating in the high-angle-of-attack, low-efficiency region, and the exit stages operating on the low-angle-of-attack side of the peak stage-efficiency point.

2. A compressor so designed that at design conditions the inlet stages operate at angles of attack lower than the angle of attack for peak efficiency and the exit stages operate at angles of attack higher than the angle of attack for peak efficiency will give only a small variation in peak over-all efficiency over the speed range of 50 to 100 percent design speed. This results from the fact that all stages operate in a comparatively good efficiency range at all speeds from 50 to 100 percent design speed. This method of design causes all the stages to operate at peak efficiency at some speed below design speed.

3. Stages having a weight flow for peak pressure ratio considerably lower than the weight flow for peak efficiency are desirable in that they result in a compressor with a better acceleration margin than stages having practically the same weight flow for both peak pressure ratio and peak efficiency.

4. Flat stage-pressure-ratio and efficiency curves give better multistage performance characteristics than peaked stage curves. Both pressure ratio and efficiency at part speed are higher for the compressor having flat stage-performance curves.

5. In every compressor there is a stage which operates over a minimum angle-of-attack range. This section of the compressor can therefore be highly loaded. The stage operating over the minimum range is usually a stage in the last half of the compressor and its position is influenced by the stage performance and the method of design.

6. Unloading the inlet stages of a compressor designed for peak efficiency at design speed by stator-blade adjustment results in higher efficiencies and pressure ratios for part-speed operation than were obtained with the original compressor. The weight flow at design speed, however, is reduced even by small changes in stator angles.

7. Loading the exit stages of a compressor by stator-blade adjustment, results in somewhat higher efficiencies and pressure ratios for part-speed operation than were obtained by the original compressor. Design-speed performance is only slightly affected by loading the exit stages.

Lewis Flight Propulsion Laboratory  
National Advisory Committee for Aeronautics  
Cleveland, Ohio

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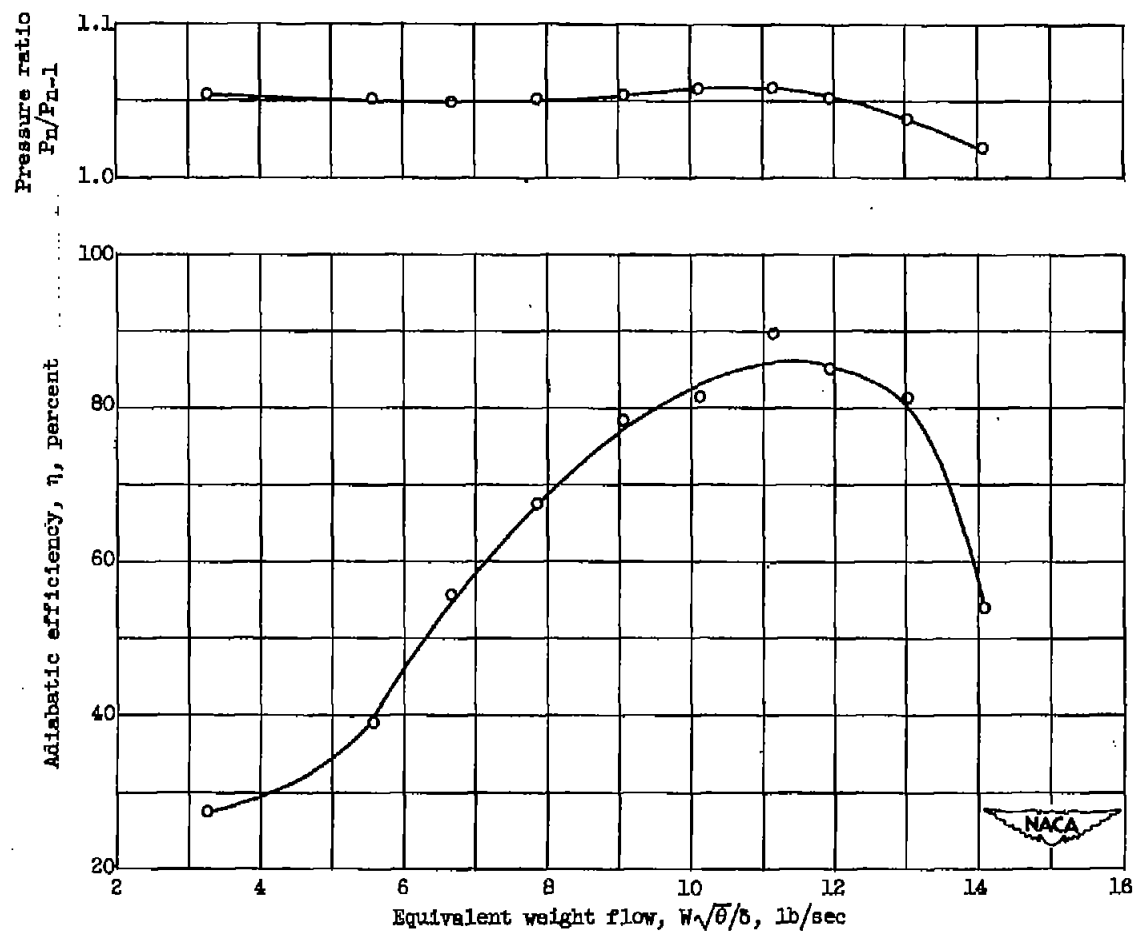


Figure 1. - Single-stage performance at 50 percent of design speed.

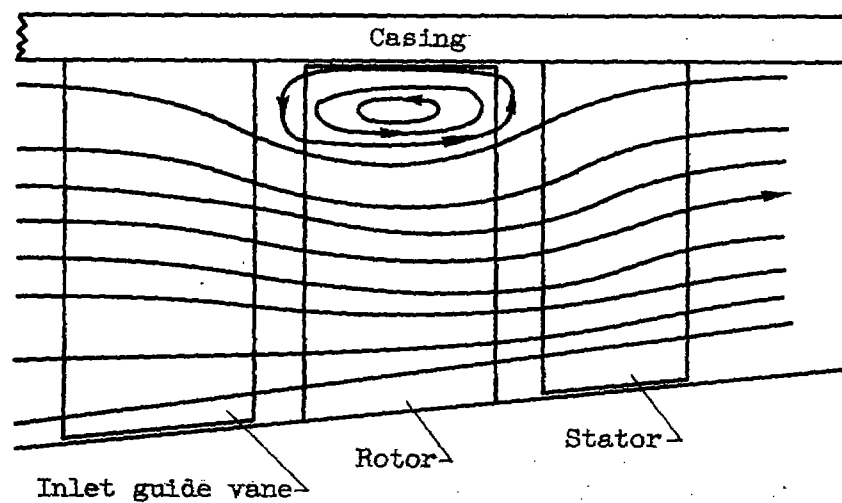


Figure 2. - Diagrammatic sketch of rotor tip-stall condition.

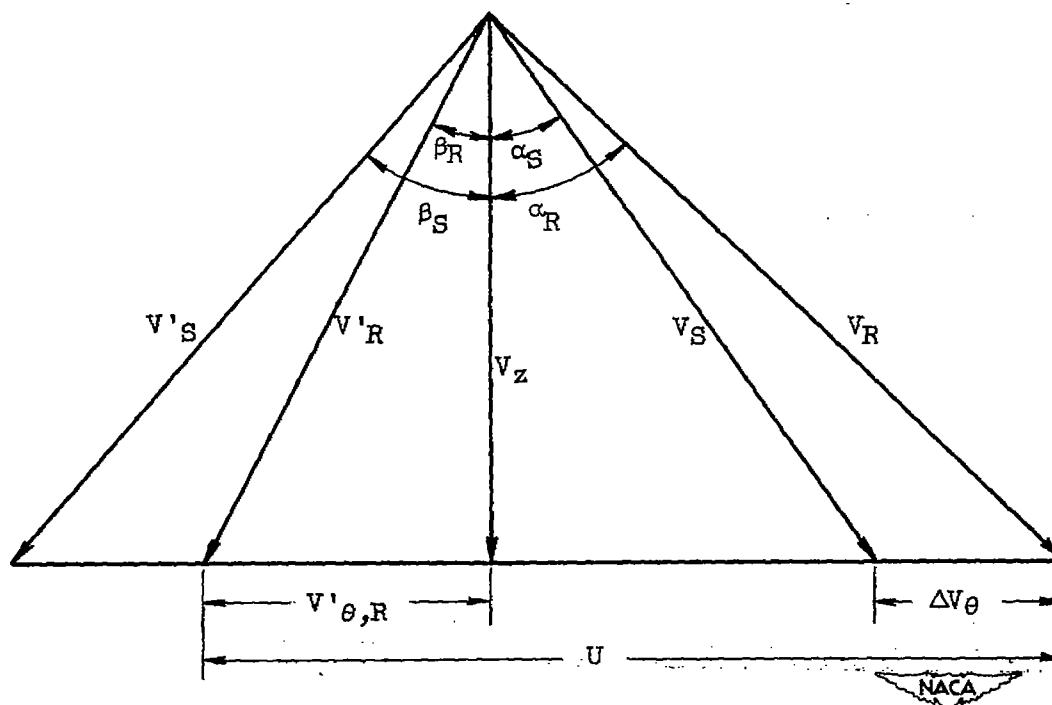
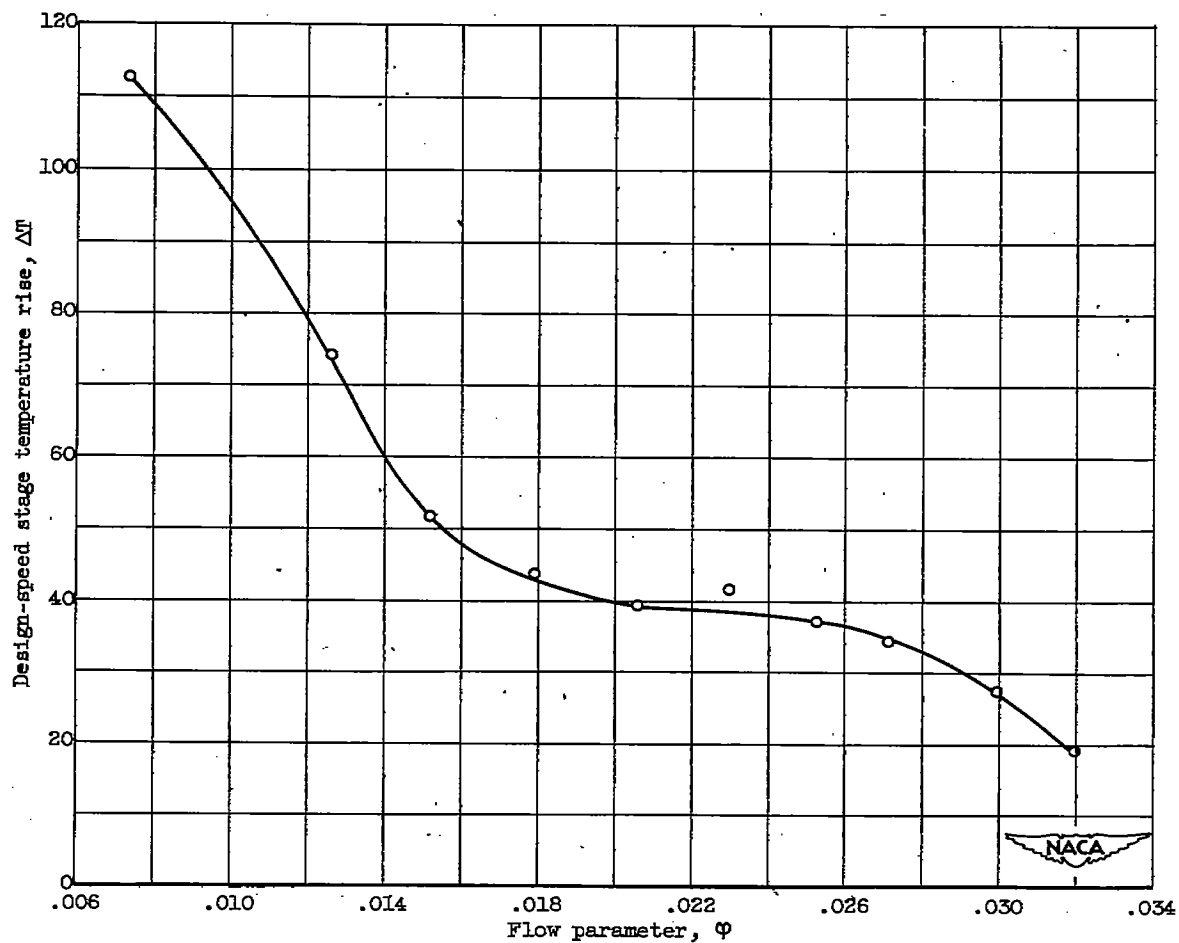
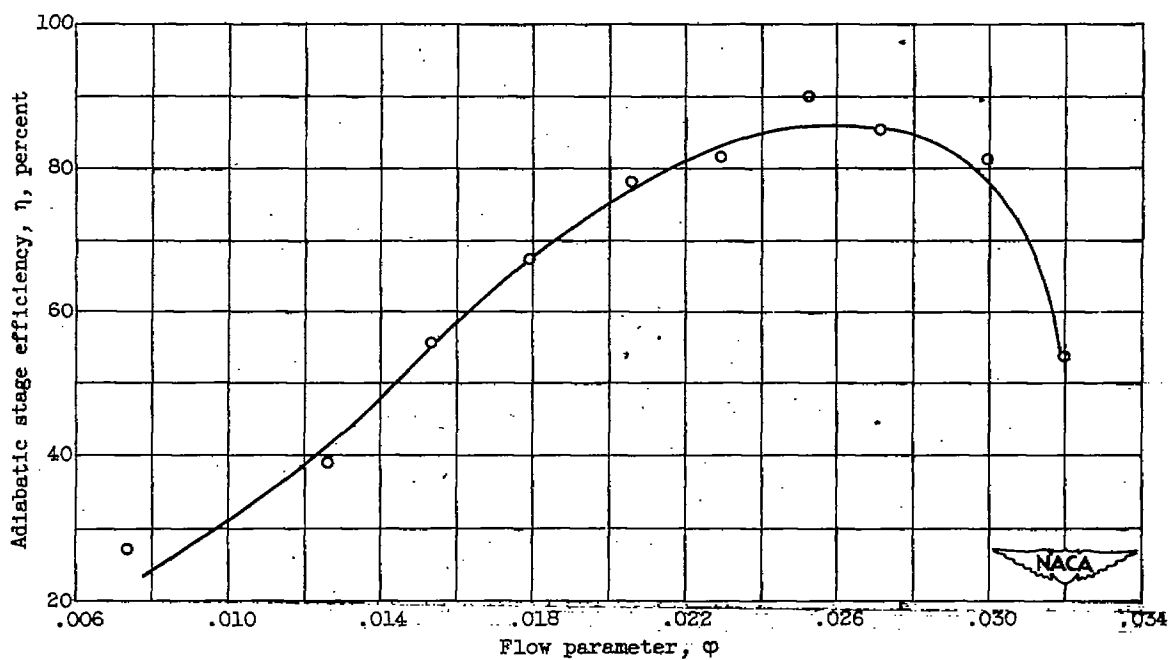


Figure 3. - Typical stage velocity diagram.



(a) Design-speed stage temperature rise.

Figure 4. - Generalized single-stage performance curve used for design I.



(b) Adiabatic efficiency of stage.

Figure 4. - Concluded. Generalized single-stage performance curve used for design I.

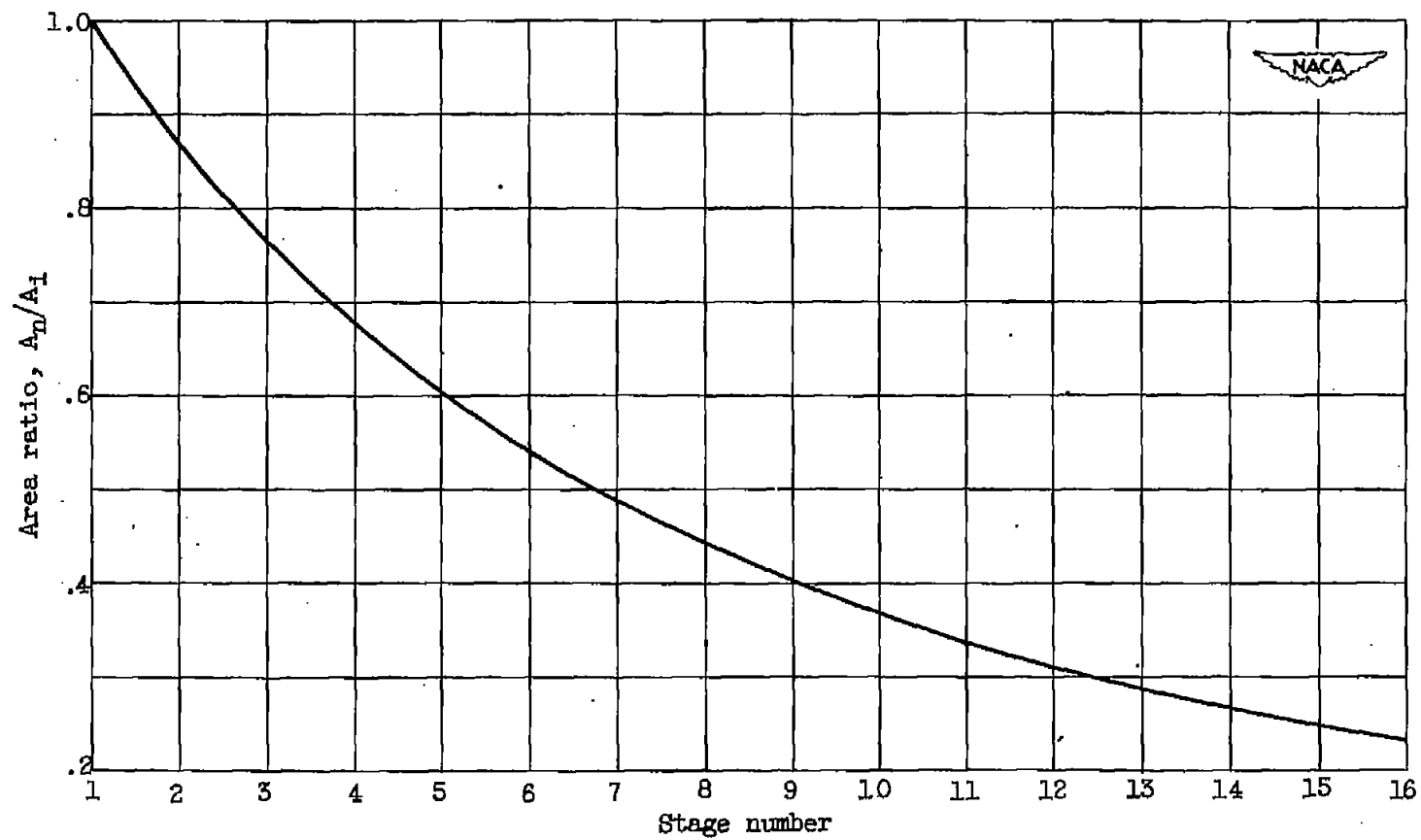


Figure 5. - Passage area variation for design I.



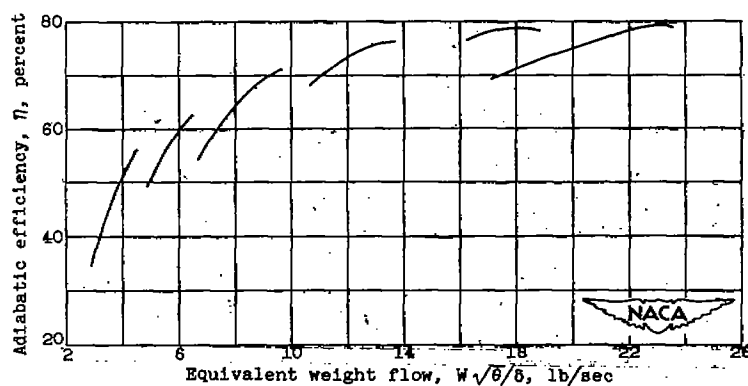
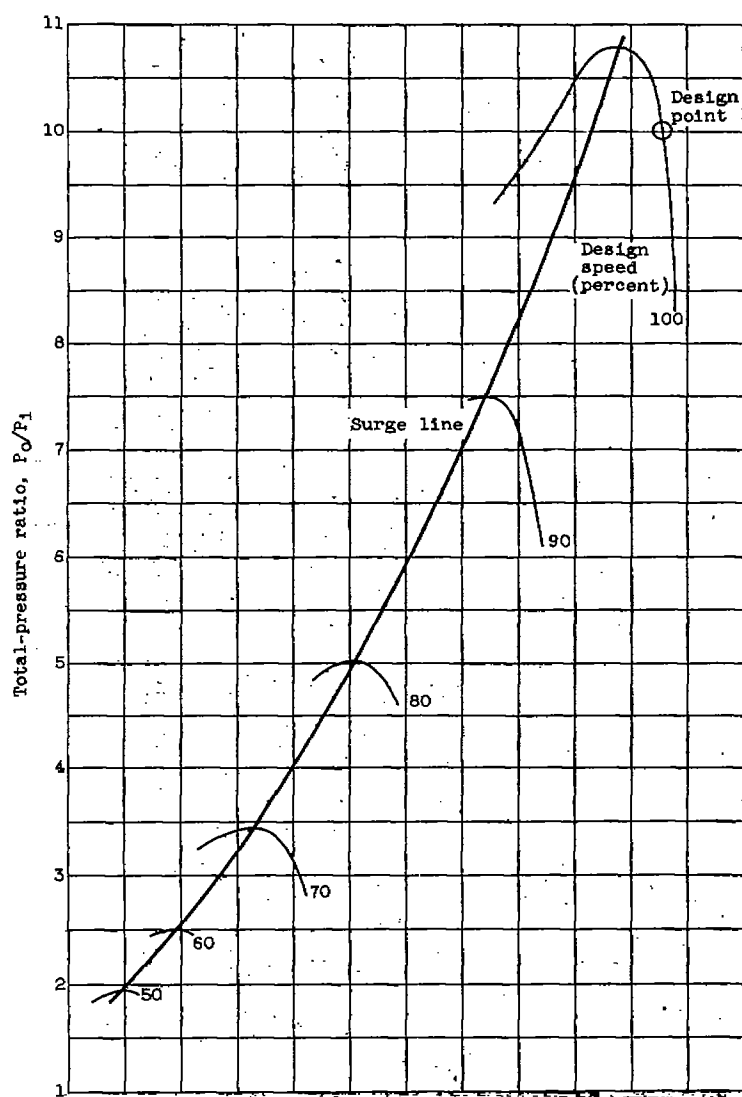


Figure 6. - Over-all performance of compressor design I at 50 to 100 percent of design speed.

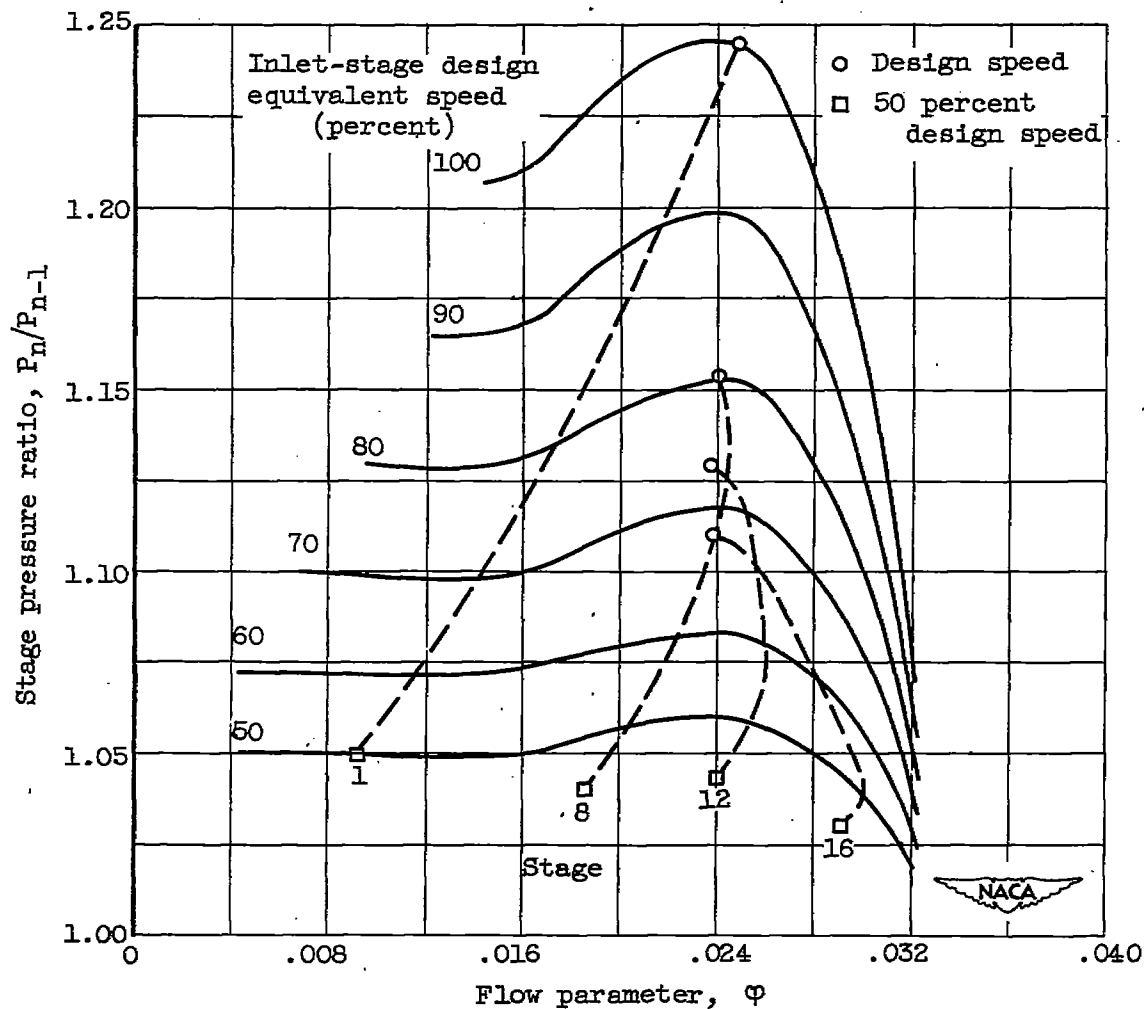


Figure 7. - Stage operation at peak over-all pressure ratio for design I at 50 and 100 percent of design speed.

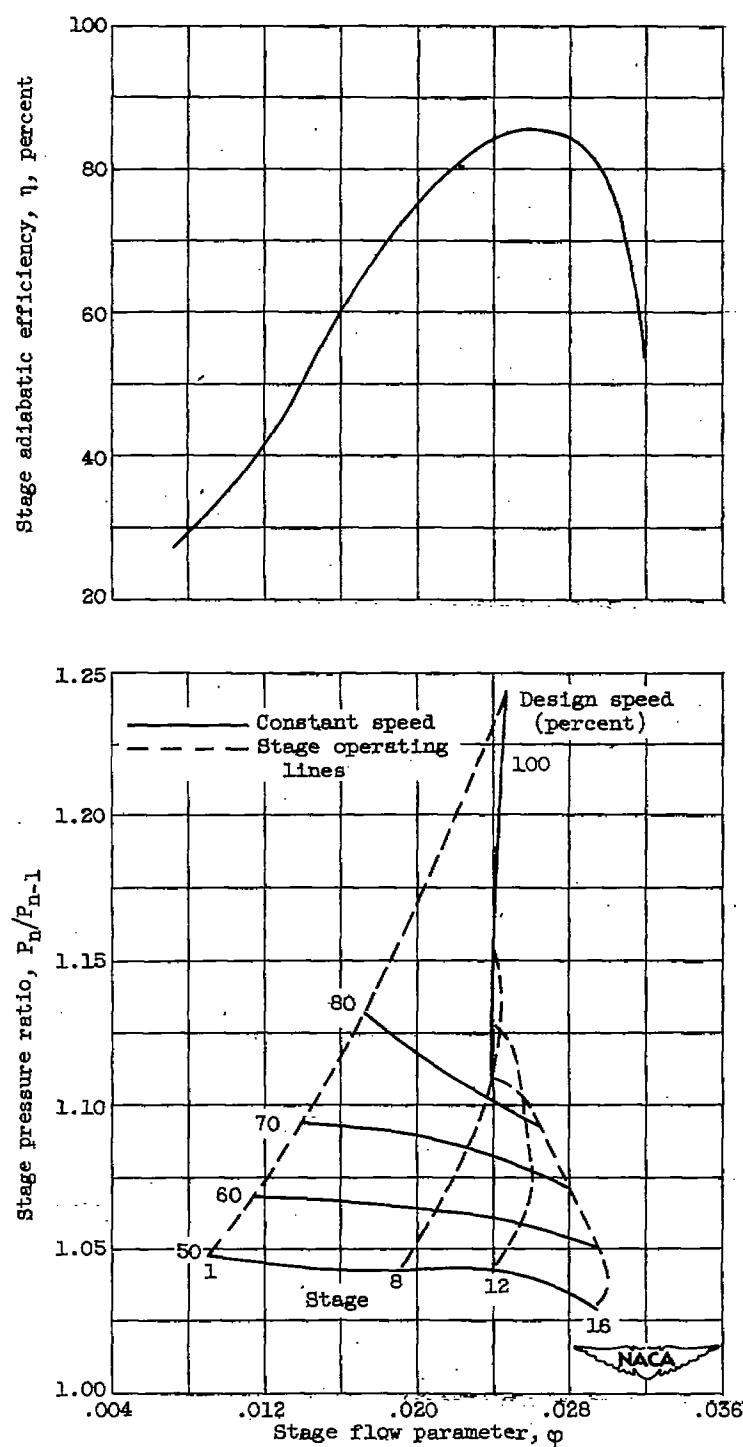


Figure 8. -- Stage operating curves for peak over-all compressor-pressure-ratio points for design I.

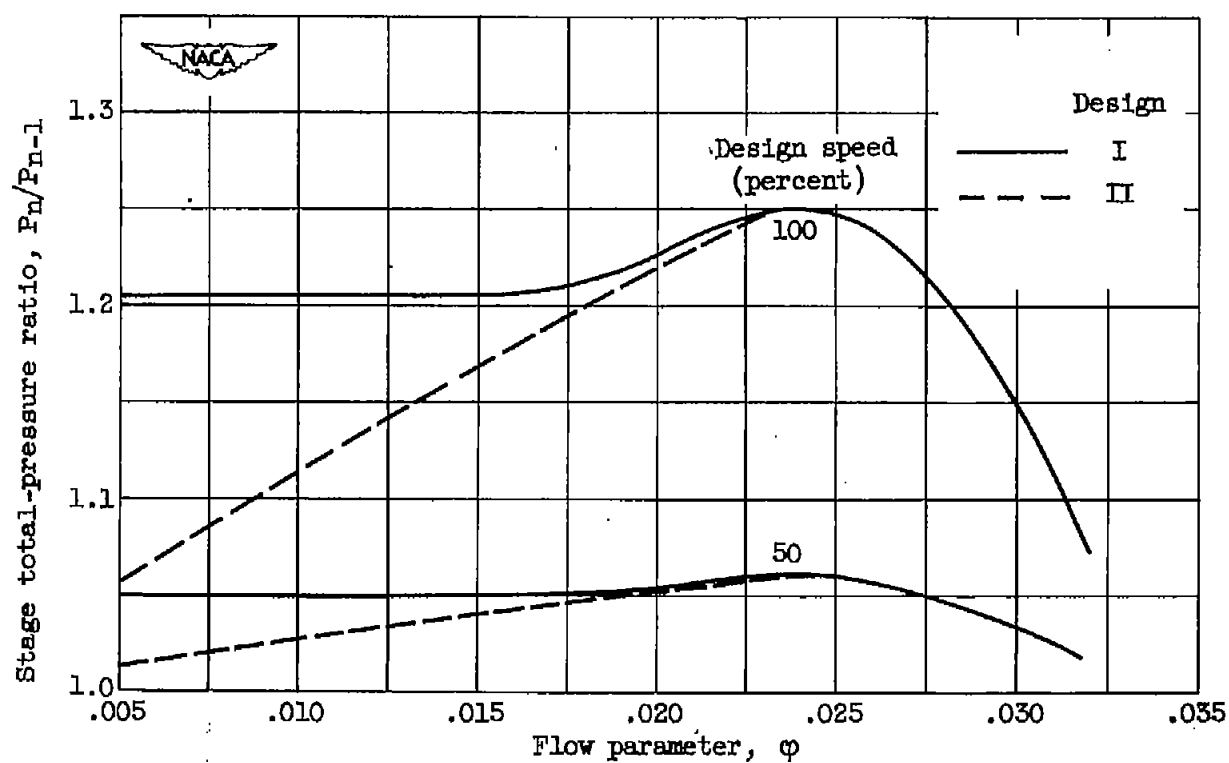


Figure 9. - Stage total-pressure-ratio curves for designs I and II.

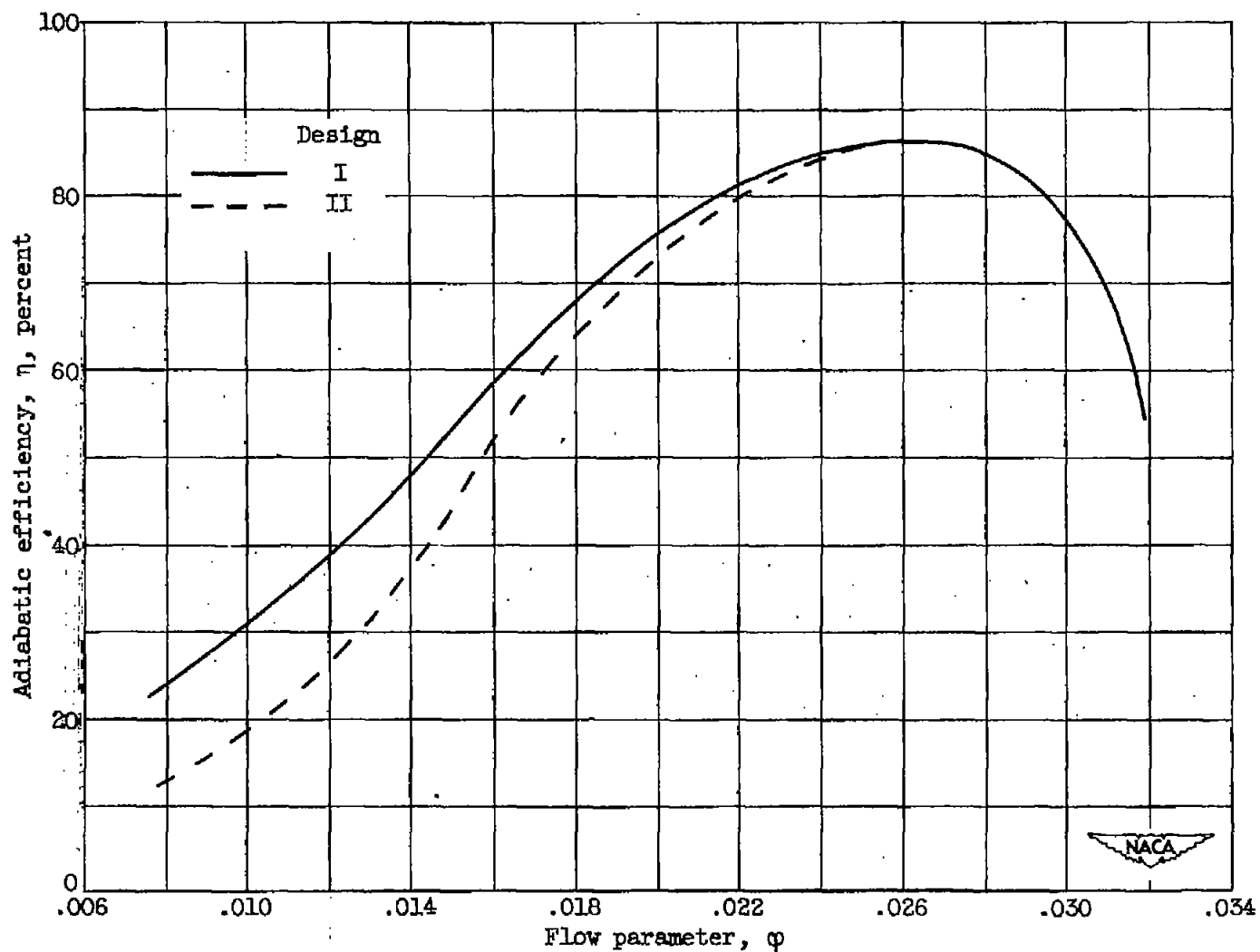


Figure 10. - Stage-efficiency curves of designs I and II.

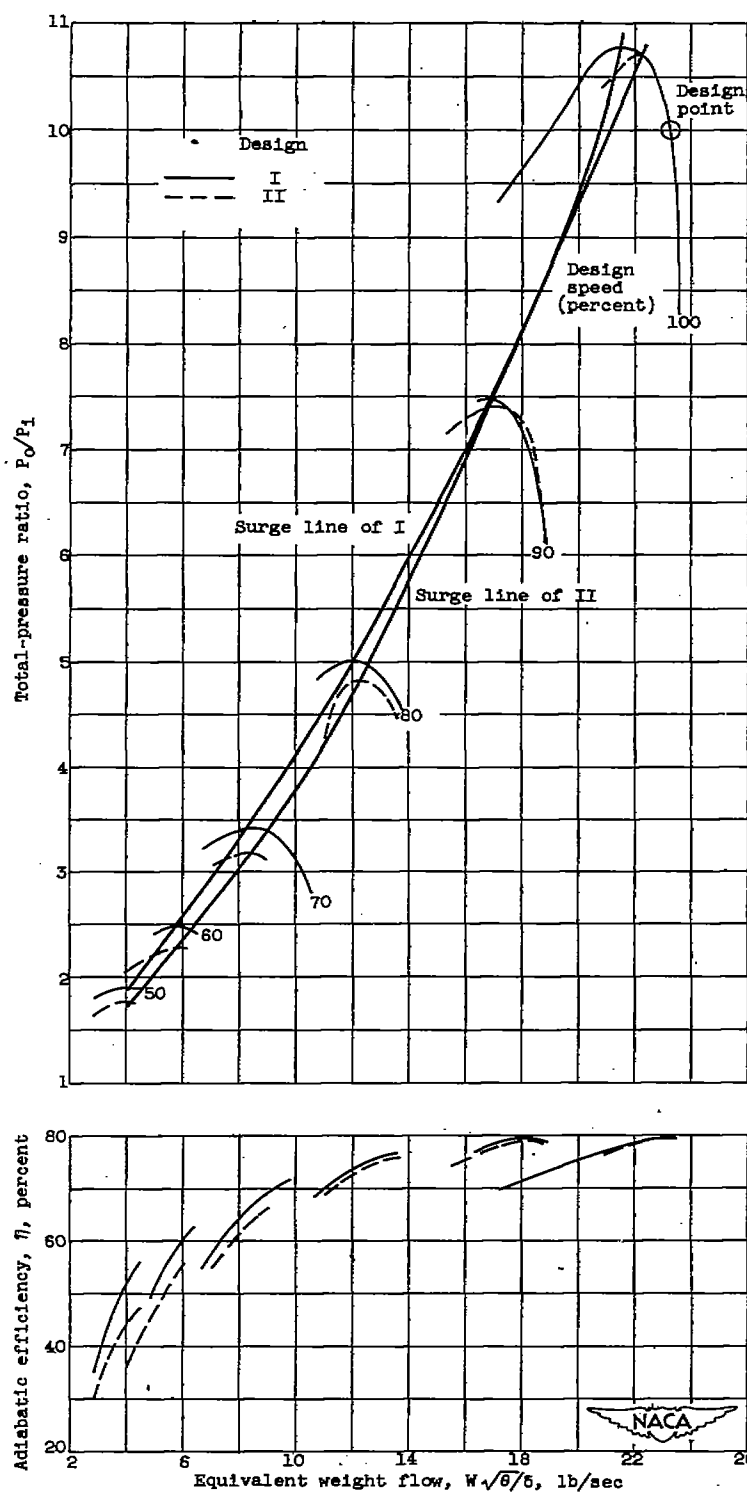


Figure 11. - Comparison of performance of designs I and II.

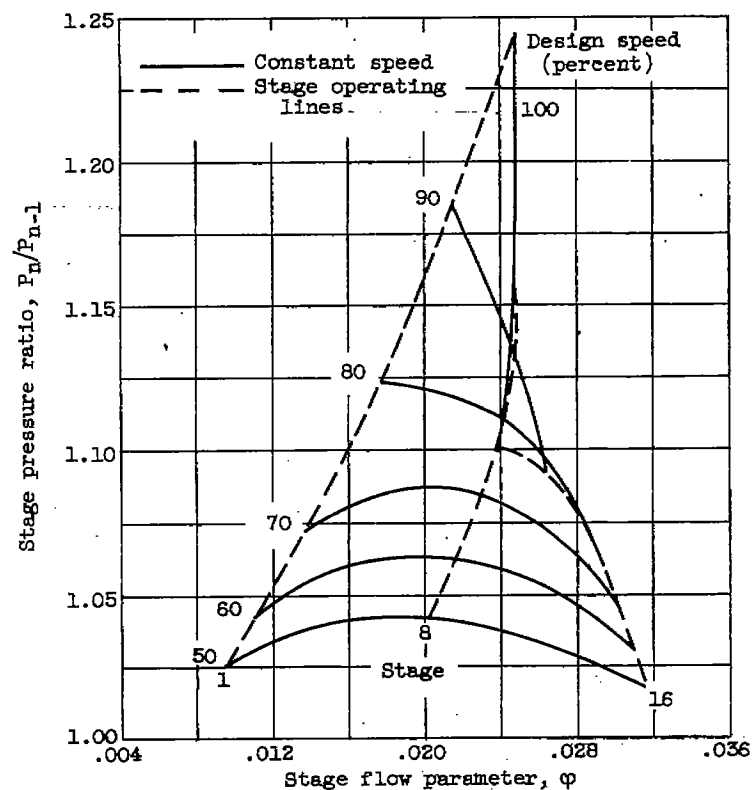
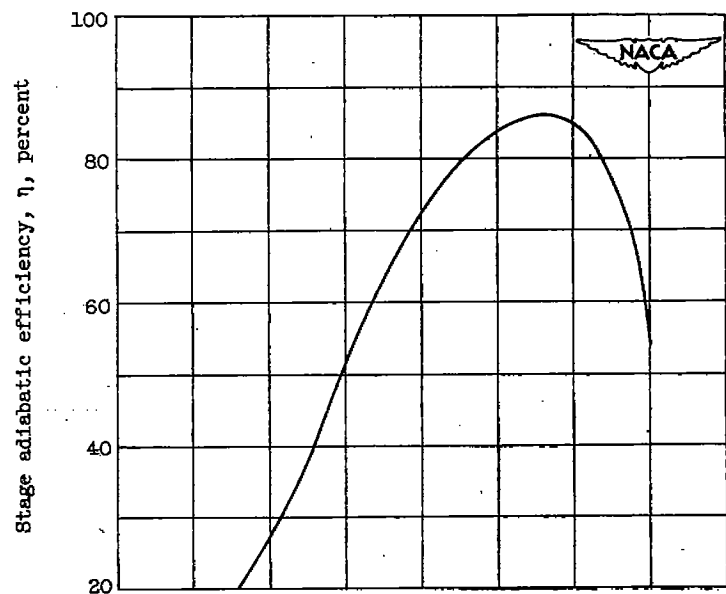


Figure 12. - Stage operating curves at peak over-all compressor-pressure-ratio points for design II.

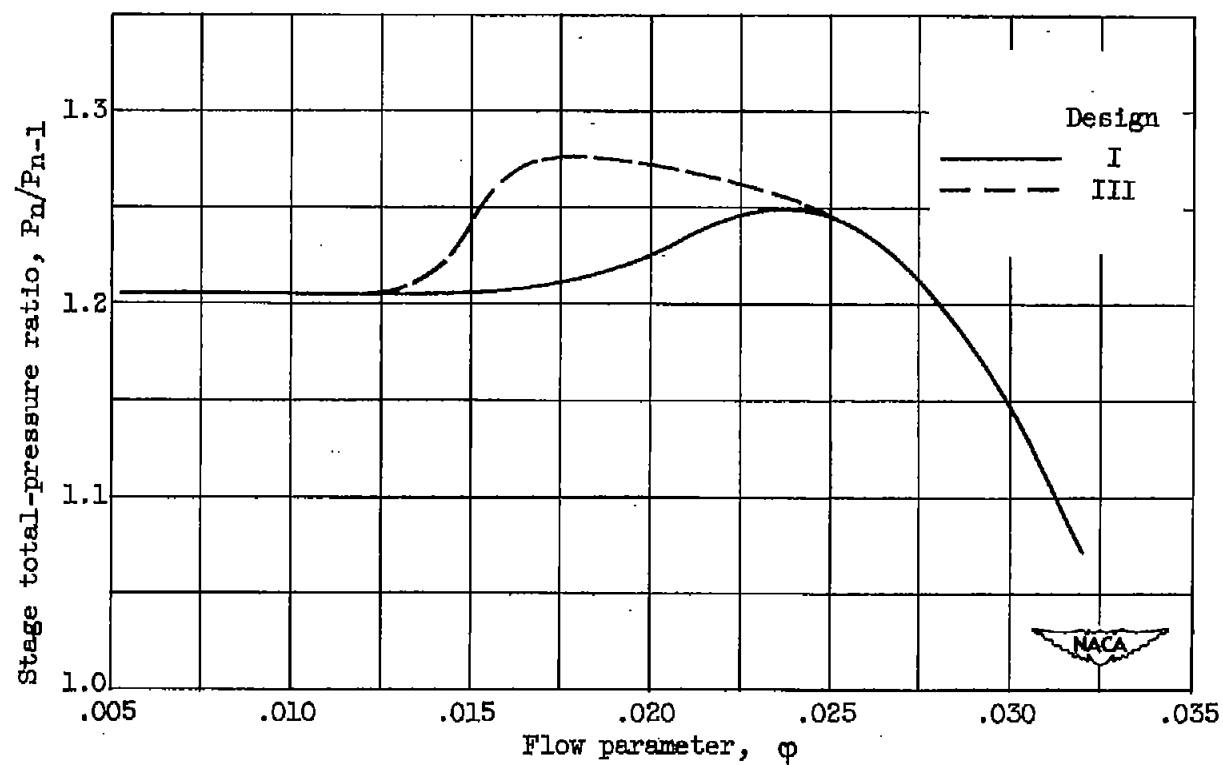


Figure 13. - Stage-pressure-ratio curve of designs I and III at design speed.



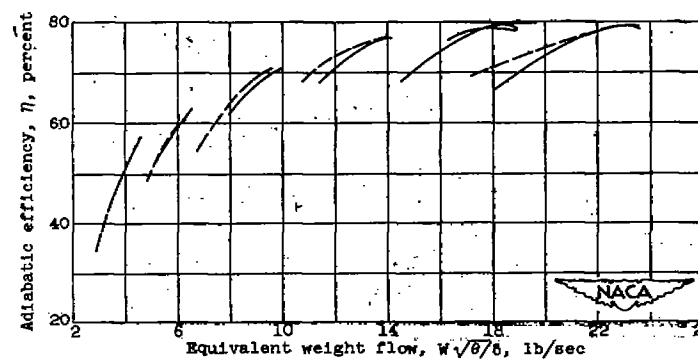
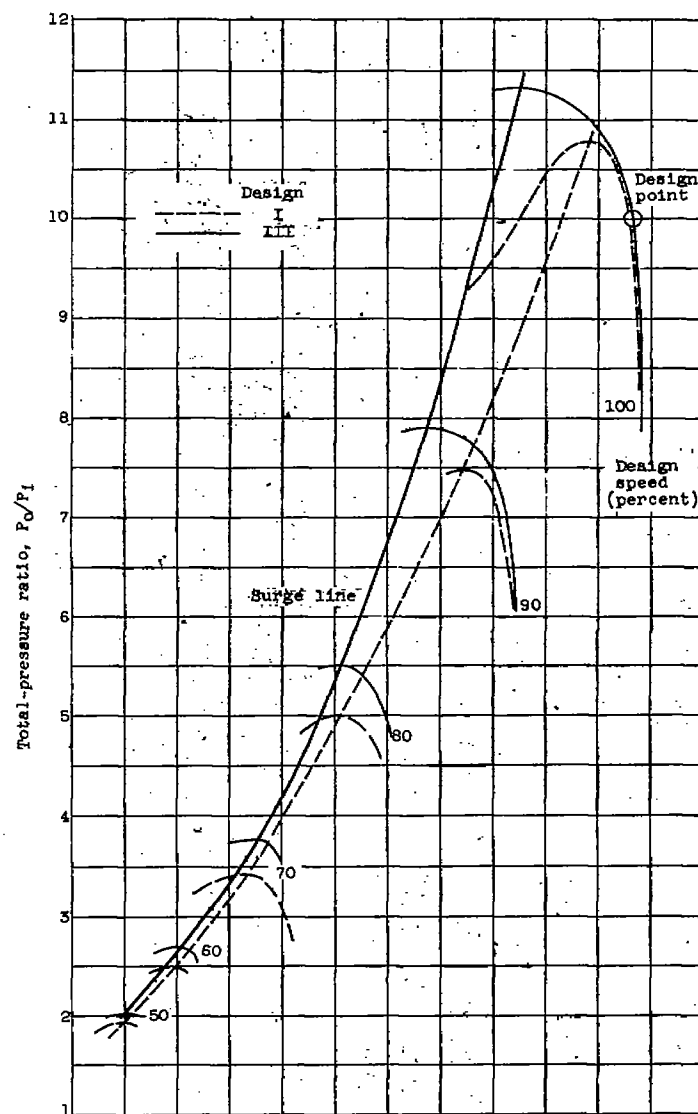


Figure 14. - Comparison of over-all performance of compressor design III with design I.

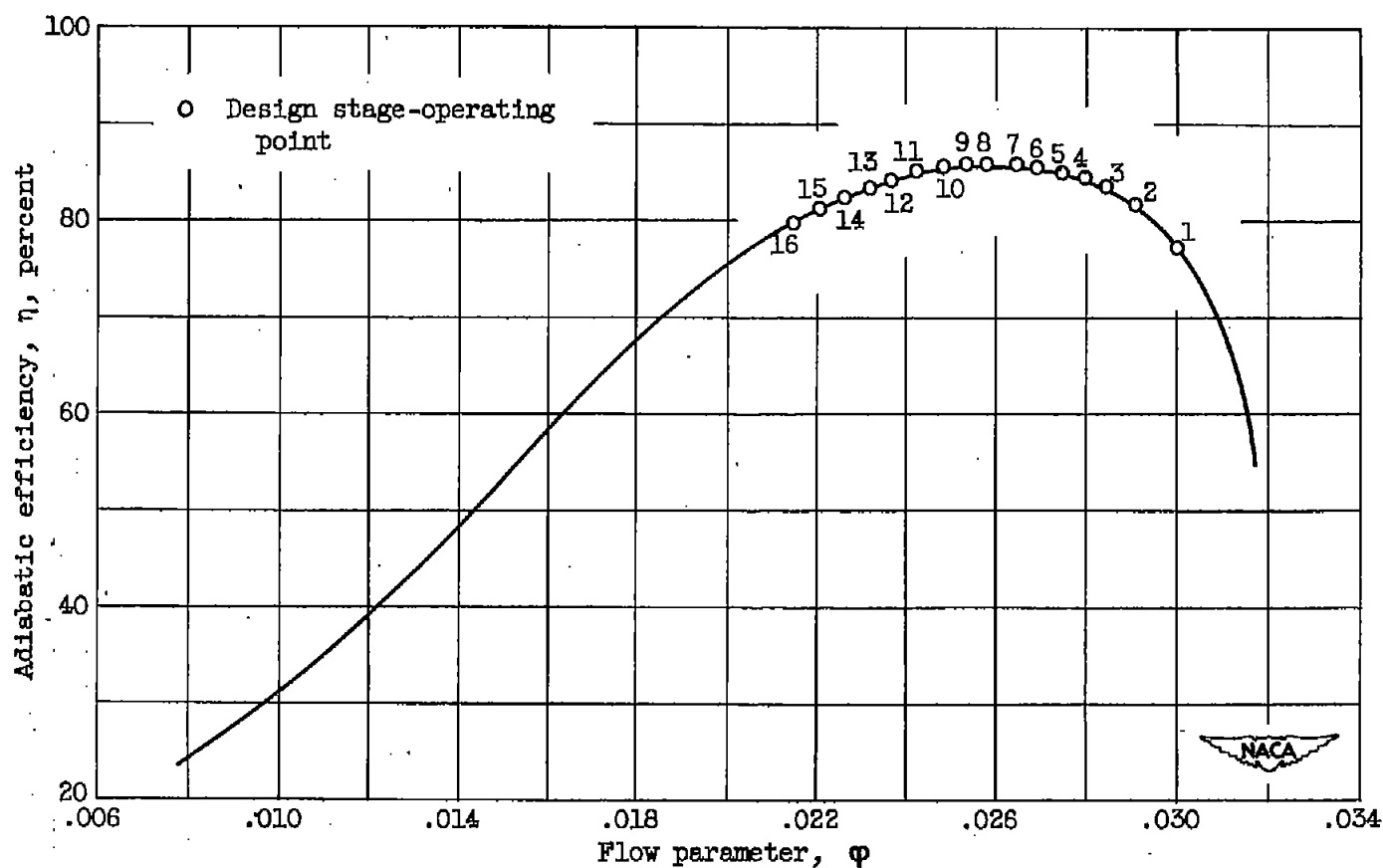


Figure 15. - Design stage-operating points for design IV.

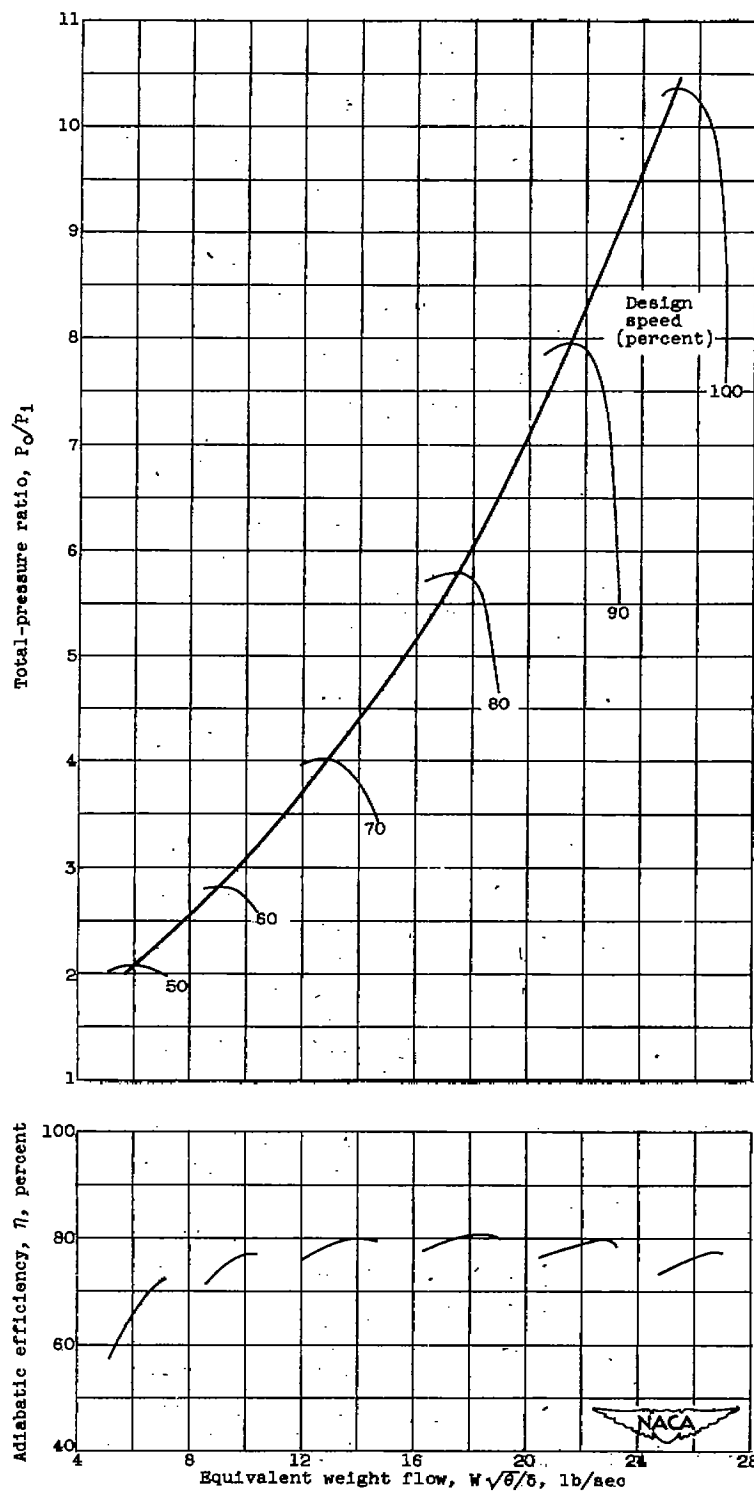


Figure 16. - Over-all performance of compressor design IV.

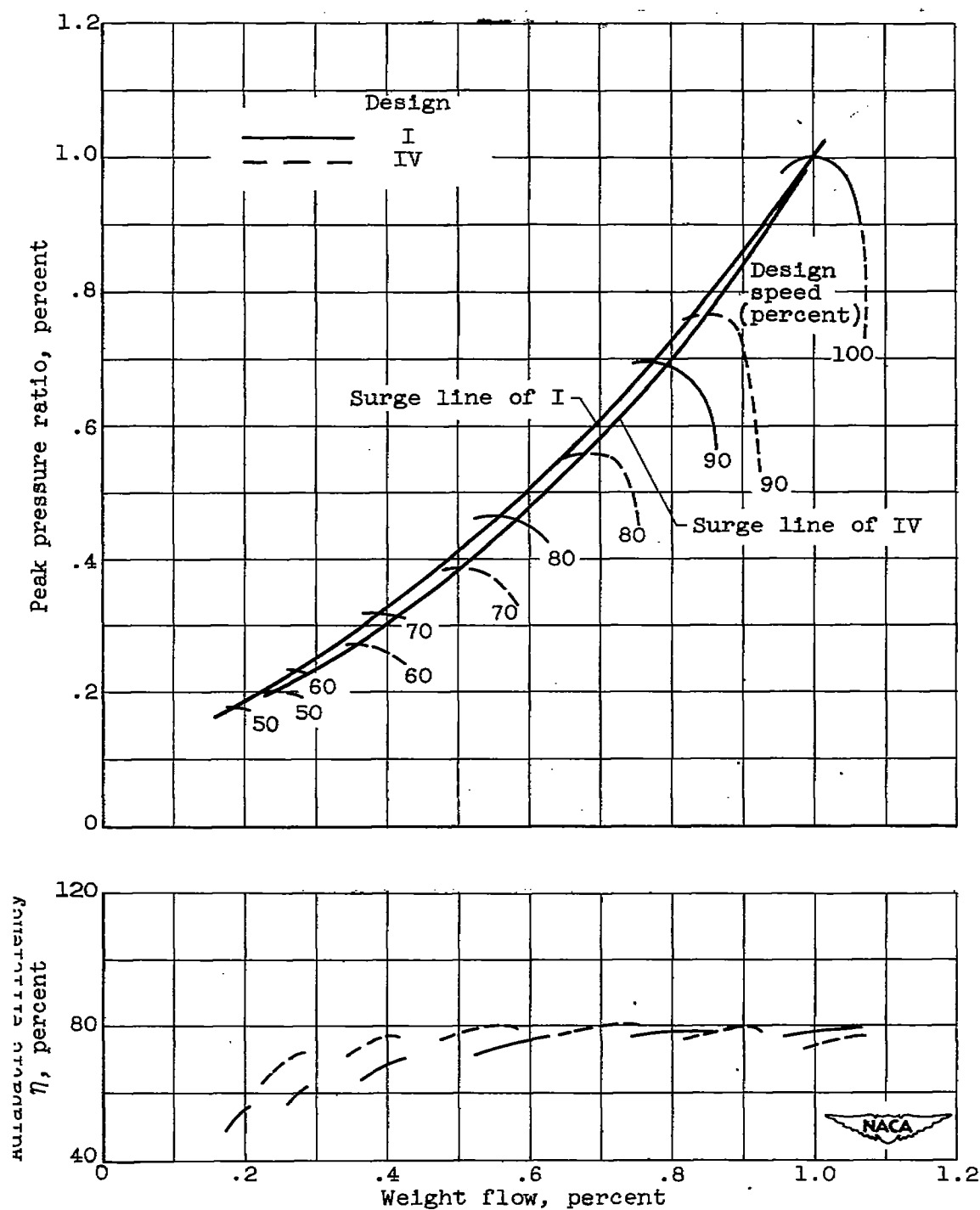


Figure 17. - Comparison of over-all performance of designs I and IV on basis of percent of peak pressure ratio and percent of weight flow at peak pressure ratio.

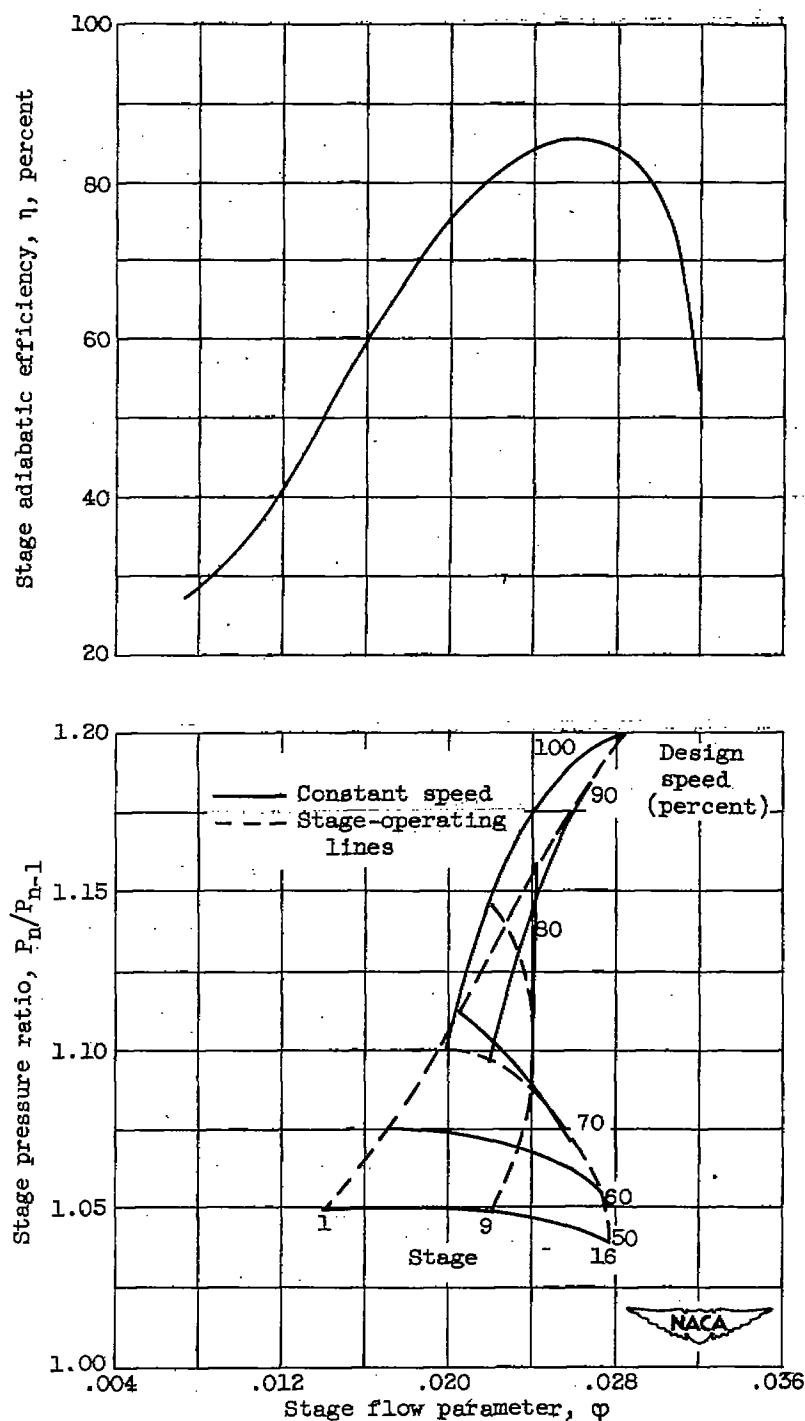


Figure 18. - Stage-operating curves for peak over-all compressor-pressure-ratio points for design IV.

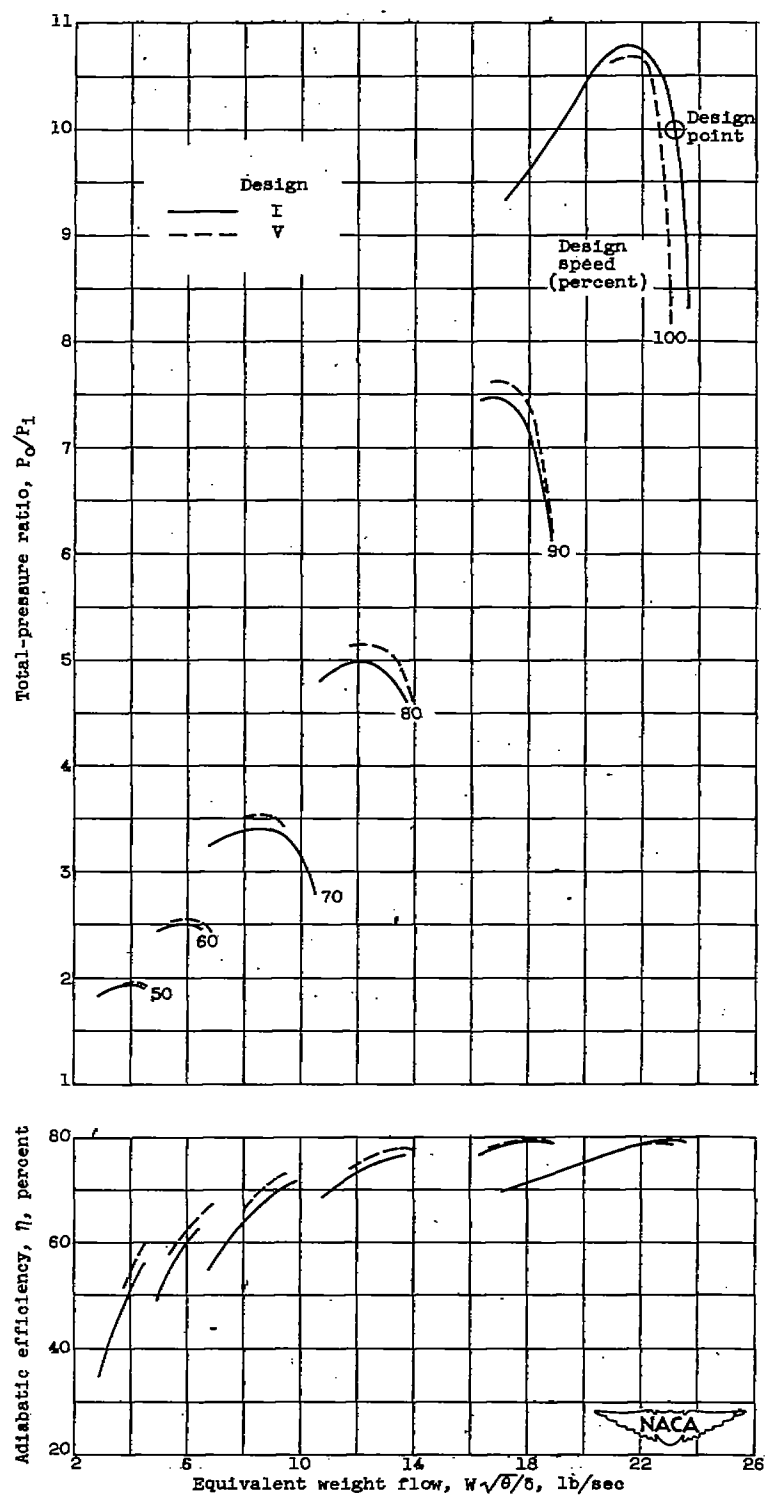


Figure 19. - Comparison of performance of designs I and V.

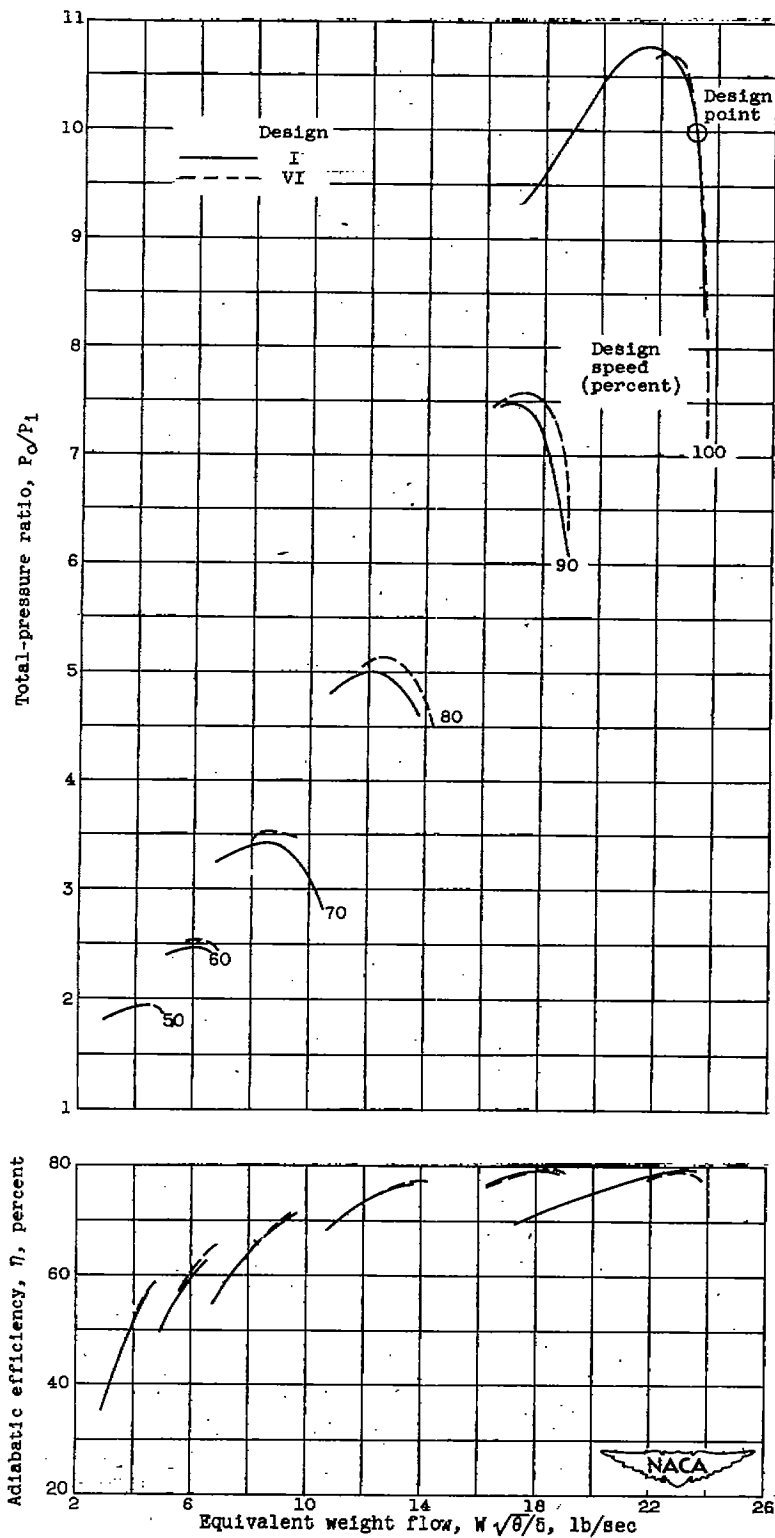


Figure 20. - Comparison of performance of designs I and VI.

# SECURITY INFORMATION

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